# Introduction to AI/ML

## Fuzzy Logic and Fuzzy Set Theory

**Dr. Tamal Ghosh**
**Department of CSE**
**Adamas University**

# Classical Set Theory

- Collection of well-defined elements that don't change from person to person or over time.
  - A={a, e, i, o, u}
- **A Family dataset:**

| Member | Age | Gender |
|---|---|---|
| Grandfather | 72 | Male |
| Grandmother | 63 | Female |
| Father | 44 | Male |
| Mother | 35 | Female |
| Daughter | 10 | Female |
| Son | 4 | Male |
| Aunt | 48 | Female |

# Example of sets

- **Universal set:** The set containing all the members of a given dataset is called Universal Set.

  U={Grandfather, Grandmother, Father, Mother, Daughter, Son, Aunt}
- **Subsets:** The set containing some of the elements of the Universal set.

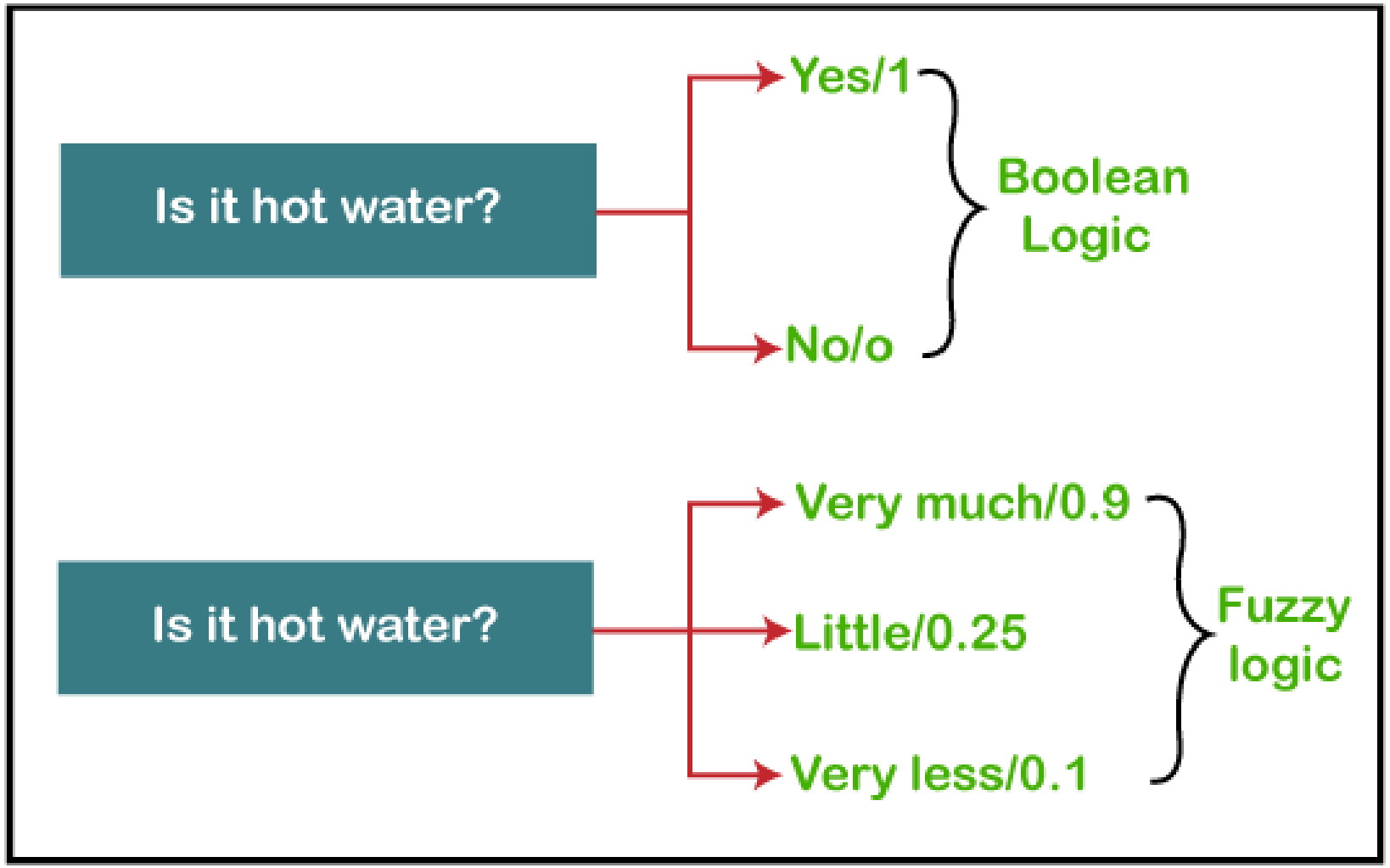
  M={Grandfather, Father, Son}

  F={Grandmother, Mother, Daughter, Aunt}

- If you are asked to create a set of senior members of the family, can you write down the set?
- ‘No’ because senior member is a ‘Fuzzy’ word.

# Another Example

- The **'Fuzzy'** word means the things that are not clear or are vague. Sometimes, we cannot decide in real life that the given problem or statement is either true or false. At that time, this concept provides many values between the true and false and gives the flexibility to find the best solution to that problem

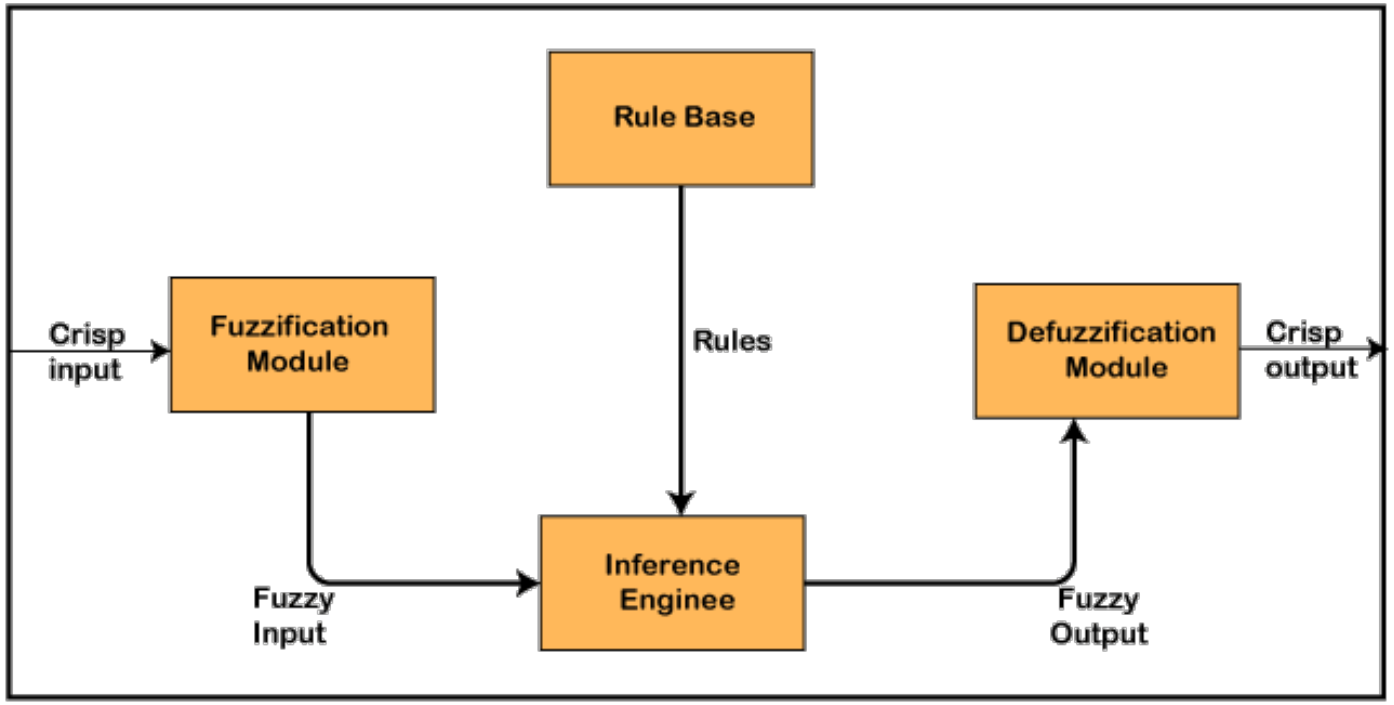


# Characteristics of Fuzzy Logic

- Following are the characteristics of fuzzy logic:
  - It is used to help the minimization of the logic created by the human.
  - It is the best method for finding the solution of those problems that are suitable for approximate or uncertain reasoning.
  - It allows users to build or create functions that are non-linear of arbitrary complexity.
  - In fuzzy logic, everything is a matter of degree.
  - In Fuzzy logic, any system that is logical can be easily fuzzified.
  - It is based on natural language processing.
  - It is also used by quantitative analysts to improve their algorithm's execution.

# Architecture of a Fuzzy Logic System

- In the architecture of the **Fuzzy Logic** system, each component plays an important role. The architecture consists of different four components which are given below.
  1. Rule Base
  2. Fuzzification
  3. Inference Engine
  4. Defuzzification
- The following diagram shows the architecture or process of a Fuzzy Logic system:



# Definitions

- 1. Rule Base
  - Rule Base is a component used for storing the set of rules and the If-Then conditions given by the experts are used for controlling the decision-making systems. There are so many updates that come in the Fuzzy theory recently, which offers effective methods for designing and tuning fuzzy controllers.
- 2. Fuzzification
  - **Fuzzification** is a module or component for transforming the system inputs, i.e., it converts the crisp number into fuzzy steps. The crisp numbers are those inputs that are measured by the sensors and then fuzzification passed into the control systems for further processing. This component divides the input signals into the following five states in any Fuzzy Logic system:
    - Large Positive (LP)
    - Medium Positive (MP)
    - Small (S)
    - Medium Negative (MN)
    - Large negative (LN)

# Definitions

- **3. Inference Engine**
  - This component is a main component in any Fuzzy Logic system (FLS), because all the information is processed in the Inference Engine. It allows users to find the matching degree between the current fuzzy input and the rules. After the matching degree, this system determines which rule is to be added according to the given input field. When all rules are fired, then they are combined to develop the control actions.
- **4. Defuzzification**
  - Defuzzification is a module or component, that takes the fuzzy set inputs generated by the Inference Engine, and then transforms them into a crisp value. It is the last step in the process of a fuzzy logic system. The crisp value is a type of value that is acceptable to the user. Various techniques are present to do this, but the user has to select the best one for reducing the errors.

# Fuzzy Sets

- A fuzzy set is a set with a membership function. The members of the set have some degree of membership or can be represented as a pair of members and membership values.
- A fuzzy set F on a given universe of discourse U is defined as a collection of ordered pairs $\{x, \mu_F(x)\}$ where $x \in U$ and for all $x \in U$, $0 <= \mu_F(x) <= 1$

$$F=\{(x, \mu_F(x)) \mid x \in U, 0 <= \mu_F(x) <= 1\}$$

- **Membership Function:** $\mu_F(x) = \begin{cases} 0, if\ x < 30 \\ \frac{x-30}{70}, if\ 30 \leq x \leq 70 \\ 1, x > 70 \end{cases}$

# Conversion of family data set

- **Fuzzy Family dataset:**

| –Member | Age | Membership | Gender |
|---|---|---|---|
| –Grandfather | 72 | 1 | Male |
| –Grandmother | 63 | 0.825 | Female |
| –Father | 44 | 0.35 | Male |
| –Mother | 35 | 0.125 | Female |
| –Daughter | 10 | 0 | Female |
| –Son | 4 | 0 | Male |
| –Aunt | 48 | 0.45 | Female |

**A={(Grandfather, 1), (Grandmother, 0.825), (Father, 0.35), (Mother, 0.125), (Aunt, 0.45)}**

# Difference between Fuzzy Set and Crisp Set

| Features | Fuzzy Set | Crisp Set |
|---|---|---|
| *Basic* | It is prescribed by vague or ambiguous properties. | It is defined by precise and specific characteristics. |
| *Definition* | It is a set of components with different membership degrees in the set. | It is a set of objects that have the same countability and finiteness qualities. |
| *Applications* | It is commonly utilized in fuzzy controllers. | It is commonly utilized in digital design. |
| *Membership* | It shows incomplete membership. | It shows the complete membership. |
| *Logic* | It follows the infinite-valued logic. | It follows the bi-valued logic. |
| *Value* | It specifies a number between 0 and 1, which includes both 0 and 1. | It specifies the value as either 0 or 1. |
| *Degree* | It defines the degree to which anything is true. | It is also referred to as a classical set. |

# Operations on Fuzzy Set

**Given Ã and B are the two fuzzy sets, and X is the universe of discourse with the following respective member functions: $\mu_A(x)$, $\mu_B(x)$**

1. Union Operation: The union operation of a fuzzy set is defined by:

$$\mu A_{UB}(x) = \max (\mu_A(x), \mu_B(x))$$

Example:

Let's suppose A is a set that contains following elements:

$$A = \{( X1, 0.6 ), (X2, 0.2), (X3, 1), (X4, 0.4)\}$$

And, B is a set that contains following elements:

$$B = \{( X1, 0.1), (X2, 0.8), (X3, 0), (X4, 0.9)\}$$

then,

$$AUB = \{( X1, 0.6), (X2, 0.8), (X3, 1), (X4, 0.9)\}$$

# Contd…

**Because, according to this operation**

**For $X_1$**

$$\mu_{A\cup B}(X_1) = \max(\mu_A(X_1), \mu_B(X_1))$$
$$\mu_{A\cup B}(X_1) = \max(0.6, 0.1)$$
$$\mu_{A\cup B}(X_1) = 0.6$$

**For $X_2$**

$$\mu_{A\cup B}(X_2) = \max(\mu_A(X_2), \mu_B(X_2))$$
$$\mu_{A\cup B}(X_2) = \max(0.2, 0.8)$$
$$\mu_{A\cup B}(X_2) = 0.8$$

**For $X_3$**

$$\mu_{A\cup B}(X_3) = \max(\mu_A(X_3), \mu_B(X_3))$$
$$\mu_{A\cup B}(X_3) = \max(1, 0)$$
$$\mu_{A\cup B}(X_3) = 1$$

**For $X_4$**

$$\mu_{A\cup B}(X_4) = \max(\mu_A(X_4), \mu_B(X_4))$$
$$\mu_{A\cup B}(X_4) = \max(0.4, 0.9)$$
$$\mu_{A\cup B}(X_4) = 0.9$$

# Intersection

**2. Intersection Operation:** The intersection operation of a fuzzy set is defined by:

$$\mu_{A\cap B}(x) = \min(\mu_A(x), \mu_B(x))$$

**Example:**

Let's suppose A is a set that contains the following elements:

$A = \{(X_1, 0.3), (X_2, 0.7), (X_3, 0.5), (X_4, 0.1)\}$

And, B is a set that contains the following elements:

$B = \{(X_1, 0.8), (X_2, 0.2), (X_3, 0.4), (X_4, 0.9)\}$

then,

$A \cap B = \{(X_1, 0.3), (X_2, 0.2), (X_3, 0.4), (X_4, 0.1)\}$

# Complement operation

**3. Complement Operation:** The complement operation of a fuzzy set is defined by:

$$\mu_{\bar{A}}(x) = 1-\mu_A(x),$$

**Example:**

Let's suppose A is a set that contains following elements:

$$A = \{( X_1, 0.3 ), (X_2, 0.8), (X_3, 0.5), (X_4, 0.1)\}$$

then,

$$\bar{A} = \{( X_1, 0.7 ), (X_2, 0.2), (X_3, 0.5), (X_4, 0.9)\}$$

# Applications of Fuzzy Logic

- Following are the different application areas where the Fuzzy Logic concept is widely used:

1. It is used in **Businesses** as a decision-making support system.
2. It is used in **Automotive systems** for controlling the traffic and speed, and for improving the efficiency of automatic transmissions. **Automotive systems** also use the shift scheduling method for automatic transmissions.
3. This concept is also used in the **Defense** in various areas. Defense mainly uses Fuzzy logic systems for underwater target recognition and the automatic target recognition of thermal infrared images.
4. It is also widely used in **Pattern Recognition and Classification** in the form of Fuzzy logic-based recognition and handwriting recognition. It is also used in the search for fuzzy images.
5. Fuzzy logic systems are also used in **Securities**.
6. It is also used in **microwave ovens** for setting the lunes power and cooking strategy.
7. This technique is also used in the area of **modern control systems** such as expert systems.
8. **Finance** is also another application where this concept is used for predicting the stock market, and for managing the funds.
9. It is also used for controlling the brakes.
10. It is also used in the **industries of chemicals** for controlling the pH, and chemical distillation process.
11. It is also used in the **industries of manufacturing** for the optimization of milk and cheese production.
12. It is also used in the vacuum cleaners, and the timings of washing machines.
13. It is also used in heaters, air conditioners, and humidifiers.

# Advantages of FL

- Fuzzy Logic has various advantages or benefits. Some of them are as follows:

1. The methodology of this concept works similarly to human reasoning.
2. Any user can easily understand the structure of Fuzzy Logic.
3. It does not need a large memory, because the algorithms can be easily described with fewer data.
4. It is widely used in all fields of life and easily provides effective solutions to problems that have high complexity.
5. This concept is based on the set theory of mathematics, so that's why it is simple.
6. It allows users to control the control machines and consumer products.
7. The development time of fuzzy logic is short as compared to conventional methods.
8. Due to its flexibility, any user can easily add and delete rules in the FLS system.

# Disadvantages of FL

- Fuzzy Logic has various disadvantages or limitations. Some of them are as follows:

1. The run time of fuzzy logic systems is slow and takes a long time to produce outputs.
2. Users can understand it easily if they are simple.
3. The possibilities produced by the fuzzy logic system are not always accurate.
4. Many researchers give various ways for solving a given statement using this technique which leads to ambiguity.
5. Fuzzy logic are not suitable for those problems that require high accuracy.
6. The systems of Fuzzy logic need a lot of testing for verification and validation.